

## G01BJF – NAG Fortran Library Routine Document

**Note.** Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

### 1 Purpose

G01BJF returns the lower tail, upper tail and point probabilities associated with a Binomial distribution.

### 2 Specification

```
SUBROUTINE G01BJF(N, P, K, PLEK, PGTK, PEQK, IFAIL)
  INTEGER          N, K, IFAIL
  real           P, PLEK, PGTK, PEQK
```

### 3 Description

Let  $X$  denote a random variable having a Binomial distribution with parameters  $n$  and  $p$  ( $n \geq 0$  and  $0 < p < 1$ ). Then

$$\text{Prob}\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \dots, n.$$

The mean of the distribution is  $np$  and the variance is  $np(1-p)$ .

This routine computes for given  $n$ ,  $p$  and  $k$  the probabilities:

$$\begin{aligned} \text{PLEK} &= \text{Prob}\{X \leq k\} \\ \text{PGTK} &= \text{Prob}\{X > k\} \\ \text{PEQK} &= \text{Prob}\{X = k\}. \end{aligned}$$

The method is similar to the method for the Poisson distribution described in Knüsel [1].

### 4 References

- [1] Knüsel L (1986) Computation of the chi-square and Poisson distribution *SIAM J. Sci. Statist. Comput.* **7** 1022–1036

### 5 Parameters

- |           |   |               |
|-----------|---|---------------|
| <b>1:</b> | N — INTEGER<br><i>On entry:</i> the parameter $n$ of the Binomial distribution.<br><i>Constraint:</i> $N \geq 0$ .                  | <i>Input</i>  |
| <b>2:</b> | P — <i>real</i><br><i>On entry:</i> the parameter $p$ of the Binomial distribution.<br><i>Constraint:</i> $0.0 < P < 1.0$ .         | <i>Input</i>  |
| <b>3:</b> | K — INTEGER<br><i>On entry:</i> the integer $k$ which defines the required probabilities.<br><i>Constraint:</i> $0 \leq K \leq N$ . | <i>Input</i>  |
| <b>4:</b> | PLEK — <i>real</i><br><i>On exit:</i> the lower tail probability, $\text{Prob}\{X \leq k\}$ .                                       | <i>Output</i> |

- 5:** PGTK — *real* *Output*  
*On exit:* the upper tail probability,  $\text{Prob}\{X > k\}$ .
- 6:** PEQK — *real* *Output*  
*On exit:* the point probability,  $\text{Prob}\{X = k\}$ .
- 7:** IFAIL — INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. For users not familiar with this parameter (described in Chapter P01) the recommended value is 0.  
*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors detected by the routine:

IFAIL = 1

On entry,  $N < 0$ .

IFAIL = 2

On entry,  $P \leq 0.0$ ,  
or  $P \geq 1.0$ .

IFAIL = 3

On entry,  $K < 0$ ,  
or  $K > N$ .

IFAIL = 4

On entry,  $N$  is too large to be represented exactly as a *real* number.

IFAIL = 5

On entry, the variance ( $= np(1 - p)$ ) exceeds  $10^6$ .

## 7 Accuracy

Results are correct to a relative accuracy of at least  $10^{-6}$  on machines with a precision of 9 or more decimal digits, and to a relative accuracy of at least  $10^{-3}$  on machines of lower precision (provided that the results do not underflow to zero).

## 8 Further Comments

The time taken by the routine depends on the variance ( $= np(1 - p)$ ) and on  $k$ . For given variance, the time is greatest when  $k \approx np$  ( $=$  the mean), and is then approximately proportional to the square-root of the variance.

## 9 Example

This example program reads values of  $n$  and  $p$  from a data file until end-of-file is reached, and prints the corresponding probabilities.

## 9.1 Program Text

**Note.** The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      G01BJF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER       (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real            P, PEQK, PGTK, PLEK
      INTEGER          IFAIL, K, N
*      .. External Subroutines ..
      EXTERNAL        G01BJF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'G01BJF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '  N      P      K      PLEK      PGTK      PEQK'
      WRITE (NOUT,*)
20     READ (NIN,*,END=40) N, P, K
      IFAIL = 0
*
      CALL G01BJF(N,P,K,PLEK,PGTK,PEQK,IFAIL)
*
      WRITE (NOUT,99999) N, P, K, PLEK, PGTK, PEQK
      GO TO 20
40     STOP
*
99999  FORMAT (1X,I4,F8.3,I5,3F10.5)
      END

```

## 9.2 Program Data

```

G01BJF Example Program Data
  4  0.50   2   : N, P, K
 19  0.44  13
100  0.75   67
2000 0.33  700

```

## 9.3 Program Results

G01BJF Example Program Results

N	P	K	PLEK	PGTK	PEQK
4	0.500	2	0.68750	0.31250	0.37500
19	0.440	13	0.99138	0.00862	0.01939
100	0.750	67	0.04460	0.95540	0.01700
2000	0.330	700	0.97251	0.02749	0.00312